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The Spirit of Calculation in the Architectural Work of Antoni Gaudí

The two main strategies of structural design related to the term "calculation" by Gaudí are the graphic static method and the hanging model method. Both methods were applied mainly to vaulting, in order to calculate compression forces, although with the graphic static method Gaudí also took tension forces in account, as we will see in the Sagrada Familia calculation. The hanging model is the superior of the two – at least theoretically – since it does not only analyze a given structural shape, but also generates the shape in a form-finding process.

Related to these calculation methods is Gaudí's sophisticated attitude towards complex geometrical and – to a lesser degree – free organic shapes in structural design. However, it is important to state that his use of complex geometry is not an integral part of his static methods (1). Gaudí's use of complex geometrical forms based on cone sections and ruled surfaces may rather be regarded as a rational way to construct his visionary structural solutions (2).

By 1800, the hanging model method, or more exact, the idea that an inverted chain generates an optimized arch shape, was well known throughout Europe via a range of architectural textbooks (3). However, in the first decades of the 19th C. many different "theories" and opinions competed about the static behavior of vaults, including bridges, and uncertainties about them were common. In order to make Gaudí's calculation methods transparent, it might be interesting to analyze some of his forerunners in the field of catenaries and hanging models. These forerunners are all German examples from1818-1840, which were recently historically analyzed. The author wants to stipulate that the cases discussed here are methodically comparable with Gaudí's use of statics, but that Gaudí did probably not know of any of them, and certainly not in detail. As we will see, the impact of the statical method on the architectural design is quite considerable and often the architect in charge had to struggle with theoretical, technical and even financial problems. All examples can be seen as mere experiments and it is highly interesting that none of them were succesful in the sense that their theories and methods were adapted by direct followers. The most probable reason for this isolation must be that the hanging model method is rather painstaking in several aspects (4).

Wilhelm Tappe and a new housing concept for the rural population (5)

The first application of the hanging model use to be discussed here is a rather revolutionary attempt, in some aspects naive, to change architecture by building houses as vaulted spaces. Wilhelm Tappe (approx. 1746-1823) was a person with a strong self esteem, a drawing expert and a building official of the Lippe County. As a scientist, he was probably a self-educated author, gathering ample knowledge by reading German and French building literature (6). His writing style shows an honest and straightforward attitude towards the theme and all kinds of social, cultural and political aspects are discussed by him, in order to boost his ideas (7). The sources of our knowledge about him are a book "Darstellung einer neuen äußerst wenig Holz erfordernden und höchsfeuersichern Bauart", edited in eight separate volumes 1818-1823, and an experimental building. Translated the title means: "Description of a new fire-proof building type, avoiding the use of wood". Tappe's intellectual isolation with these studies can be seen from the fact that he had paid the edition of the publications himself, the illustrations being printed in lithography, instead of the improved technology of engravings (8). The experimental house, like Fig. 16 in Tappe Volume 1, was built in 1818 for Fürstin-Regentin von der Lippe. Another case was a mill built in his manner. In his publication Tappe points out that the building turned out to be relatively cheap and the user was satisfied, stating this with favorable letters.

The main goal of Tappe with his new architecture seems to be political. Around 1820, Germany was in poverty, only recovering slowly from war and yet not fully participating in the industrial revolution, like France, England and Belgium. The rural population needed cheap housing urgently. Tappe hoped to help resolve this problem by his new house type (9).

The basic shape of Tappe‘s new house type was a high cupola on a circular plan. These houses had an interior span of 15-30 foot (approx. 4.5 – 9 m), the bigger being subdivided by walls. Next to this he offered a big variety of buildings with barrel vaults of a similar shape and monumental arches. In those days the perfect statical shape of an arch was determined on the basis of experiments. The raised arch types, like high proportioned ellipses, parabola, pointed arch and, of course, catenary, turned out to result in less thrust forces than for instance the half-circular arch. The section of Tappe‘s house type was either a pointed arch of the gothic type or elliptical. In relation to Gaudí’s designs it is important to know that Tappe – like a range of international authors in his times – was uncertain about the right shape and that he preferred the ojival or elliptical shape over the catenary. As a reason for this choice instead of the catenary, he wrote that measurement and reproduction of that shape was rather difficult for poorly trained builders, like the rural users themselves (10). This is an important issue which turns out to be one of the main design problems of this kind of vaulted architecture and which Gaudí resolved in various ways. Tappe on the other hand, like many others, rejected the catenary shape for esthetic reasons. The catenary shows an oblique line to its end, whereas the ellipse and the pointed arch have a vertical tangent in the meeting point of arch and soil. The catenary shape was deemed to be archaic and primitive in a cultural sense.

Tappe, understanding that only the catenary could be considered theoretically sound as a model for an optimized arch, tried to show that by changing the weights of a chain a pointed arch or an ellipse could also be ”easily“ produced. In his assumption that there was the potential to influence the catenary shape he made the mistake of not proving it by experiment. If he would have done the experiment, he would have seen that a chain held by two distant end points can in no way have a vertical tangent like the ellipse or a pointed arch (11). From his false assumption, Tappe drew further erroneous conclusions about the shape and meaning of a catenary.

Why is Tappe important in relation to Gaudí‘s hanging model method? It is important that anybody involved with this innovative subject, even in our times, acknowledges that it is quite a difficult theme with a great potential for errors of a theoretical and experimental kind (12). Tappe demonstrates this in a rather tragic way, but also, unconscious of his half-knowledge, he finds the beginnings of a new architecture which positively shows what one could call ”proto-Gaudinism“. Comparable characteristics of Tappe‘s and Gaudí‘s work are tall cupolas and arches, a tendency towards organic form, a structural and architectural design departing from statical reflections (13).

Heinrich Hübsch and an optimized Neo-Romanesque church (14)

In the architect and architectural writer Heinrich Hübsch (1795-1863) from Karlsruhe we recognize an enlightened personality, who added transparency to the architectural debate of the German Romantic period. Internationally famous as the writer of the architectural pamphlet ”In welchem Style sollen wir bauen?“, Karlsruhe 1828 (15), Hübsch advocated the so-called Rundbogenstil, of Romanesque or Renaissance shape, in a certain opposition to emerging neo-gothic results. As a pupil of Weinbrenner, his working method was as accurate as it was productive. In his theories Hübsch was very keen to start from built architectural examples, of which he had published numerous measured drawings, often from Byzantian and early Christian works in Italy. These illustrations surmounted the precision level of his time and he grew to be an expert in structural design. On the other hand he was interested in inventive techniques of early 19th C., be it the possibility of lightweight steel structures or the hanging model method, of which he published short but clear enough descriptions. All this resulted in lightweight buildings of high quality, including many churches and public buildings.

Of several designs with the hanging model method, the catholic church of Bulach (1828-1837) near Karlsruhe was executed and it still stands soundly in all its natural beauty. Hübsch‘ interpretation of the hanging model principle was both simple and consistent. By trial-and-error Hübsch experimented with a suspended string, at regular distances loaded by weights and thus simulating building parts. Behind this hanging model he had drawn the section of a vaulted structure.

In the case of the Bulach church he concluded that the mere natural chain shape, the true catenary without any additional weight, was practical to fix the proportions of his church design. This is an important design approach, since many errors derive from the urge by architects to dictate the design form to the model, which is in contradiction to the formfinding process of hanging models. In order to describe the three-nave system, Hübsch suspended a chain from two distant points, simulating the buttresses in the exterior walls, and simulating the pillars in its central part, held by two straight strings.

The three-nave church design with a 60 foot interior width (approx. 18 m) was interpreted as an additive system of equal vaulted parts, of limited depth. He could be sure that in longitudinal direction the structure would be safe with the limited depth of these parts (approx. 4.5 m), since only small thrusts would occur, surely smaller than the thrust forces in transversal direction. One such building part consists of two exterior walls with small buttresses near the ground (16), two octagonal pillars, inclined barrel vaults in the side bays, two barrel vault sections, leaning to each other in the central bay, and roofs in wood on the central and side bays. Of high importance are additional elements which show the creativity of Hübsch, who needed the additions to keep the architectural shape in line with his self-proclaimed ideal style, being Romanesque. The additional elements are on the one hand ”flying buttresses“ in the roofs over the side bays. These flying buttresses were needed to match the structural shape given by the thrust line of the hanging model. On the other hand Hübsch did have a problem with the shape of the transverse arch connecting the pillars. This half-circular arch did not match the shape of the chain’s top part well, and therefore he added material in order to avoid bending forces which occur when the thrust line is outside the material zone. His eminent structural approach can be demonstrated by the specific shape for these additional elements in the roof space of the central bay. By building these volumes with carefully distributed voids he managed to avoid additional weight, which would disturb the regular weight distribution as dictated by the hanging model.

An interesting aspect is that the pillars are vertical, which can be considered a tribute to ”normal“ architecture and which did not coincide exactly with the slight inclination in the hanging model. With a confident structural approach, Hübsch argued that he had added some extra material on top of the piers, which would havea positive influence on the thrust line.

The scientific approach of Hübsch is also proven by the fact that he built a test structure, at half-scale of the building. He published the result in Bau-Werke 1838/1852. The stone structure was a simple two-dimensional plane arch, some 10 m high and 45 cm deep, with a weight distribution like the hanging model. Loading of his test structure under several weight conditions showed an additional stability. One of the effects of the test structure for Hübsch was that the hanging model theory was confirmed (17).

Hübsch was eager to build ”natural structures“ which obey the laws of nature. This kind of thinking in early 19th C., much in common with current notions of ecology, was extremely rare in the international cultural world (18). Of course Gaudí should be considered in the same way of thinking, as proven by many of his statements (19) and moreover by his work. But in Hübsch we recognize the concise designer in both architectural and structural sense, with equal interest for the general statical approach as well as proper detailing. Such professional behavior is similar to the design and building processes led by Gaudí, who was sceptical about theoretical generalizations, which exceed the specific building case.

Carl-Anton Henschel and a lightweight foundry cupola (20)

Around 1810 in the city of Kassel, Carl-Anton Henschel (1780-1861) started a new foundry, which would later become the Henschel-Hanomag industries (21). The personality of Henschel is puzzling in the variety of his knowledge, inventive creativity and interests (22). Starting as a mine engineer, Henschel worked as industrialist, railway producer, inventor of all kinds of machinery, architect and philosopher. His artistic working field was enforced by the fact that the foundry had commissions for sculptural and decorative works, which often were designed by his younger brother, Werner Henschel. A philosophical booklet by Henschel is ”Zur Aesthetik der höheren Baukunst“ (Kassel 1850), in which he pleads – based on a positivist rationalism - for natural structures, void of the burden of stylish elements (23). His architectural activity is demonstrated by the house for the family, with an artist studio in the attic for his brother. The house, next to the foundry which will be discussed in relationship with the hanging model, was elegant and of a time-less charm, as we can see on an old photograph and other detailed illustrations.

In 1837 the Henschel firm lost its wooden roofed foundry by a fire and a new foundry was urgently needed. Henschel designed and built the foundry himself in 1837 since the Henschel firm owned a brick factory. The foundry had an exterior appearance reminiscent of the Roman Pantheon (24). The oculus was covered with a light metal roof. A long one-story atelier with two chimneys completed the functional building.

In the description of the foundry one has to distinguish two different designs by Henschel, the first was the design for the intended execution, which in the second design had to be changed slightly as the consequence of a collapse of part of the cupola. The design method is reported to be based on a hanging model (25). In order to keep the description clear I will mainly refer to the first design.

The extreme lightness of the foundry building becomes clear from the surviving design drawing. Any civil engineer or architect with structural interests is astonished about the ratio of approx. 18 cm up to 32 cm cupola thickness to a 16 m interior span. Even stranger is the fact that the cupola thickness not only – as one might expect – reduces from the base upward, but also increases again near the top, in the opposite direction. An analyzis of the cupolar shape shows that the section consists of a pointed arch, with an opening in the top and with an oblique tangent in the base. Since the pointed arch consists of two exact circle segments, some disbelief about this design is justified, made after the formfinding method of a hanging model.

However, the reconstruction of the hanging model proved that this geometrical form really was derived from a hanging model (26). Why did Henschel search for the ordinary shape of a circle segment with a hanging model, which normally tends to a ”free shape“ like the catenary? This question can be plausibly answered, since Henschel needed a fast way to get this cupola built. With a simple enough device, which we reconstructed schematically, he could build the cupola without needing any scaffolding, by laying the bricks in circular rows, which in a ring direction tend to push together and thus keep the masonry in situ. Such a device, in which the end of a pole describes a constant circle radius in vertical direction and a varying circle radius in horizontal direction, enables the bricklayer to find the position of any brick in the void space. At least theoretically, this assumption was correct, but, as we will see, some problems still arose during construction, including a collapse.

How did Henschel manage to generate an exact circle segment for the building's section with the hanging model method? As we think to have understood from the reconstruction work, Henschel experimented with different shapes, until he found a “free shape“, being quite close to a circle segment. This cupola shape had statically favorable form characteristics, like an oblique tangent at its base and a slightly pointed top (27). The rest Henschel had to do was to match the desired thrust line of the geometric shape by correcting the weights in the model. Since the width and height of the cupola elements were fixed he could only vary the thickness. Such a rather painstaking process is mathematically defined as an iteration process, step-by-step getting nearer to coherence (28). Henschel resolved the problem of varying thickness in a logical way, which one may call an “onion-peeling-method“. The cupola consists of cylindrical bricks, open to the interior, and with an external covering of one inch of cement as a weather tight skin (29). Henschel did vary the cupola thickness only on the intrados for technical reasons concerning the watertightness. In order to keep the number of different lengths of the tubes as small as possible, he chose seven lengths between 6 and 12 inch (0.5 foot - 1 foot), with steps of one inch. In this way he could simply add or take off weight at any height level of the cupola and translate this in the hanging model.

The thrust line, derived from the hanging model, was probably chosen at a distance of 4 inch from the cupola extrados, equal to 1 inch cement cover and 3 inch of tubular brick. In this way he could be sure that in the thinnest, and thus weakest, part of the cupola the thrust line was inside the middle third space, which is considered to be safe by statical theory.

The cupola collapsed during construction, luckily without hurting people (30). Our analysis, with the help of the hanging model method as well as the computer calculations by Tilman Hörsch, has hypothetically shown why the collapse occurred and how Henschel solved the completionof the cupola without changing his design too much. The collapse probably occurred because the ring-compression gradually increased to a critical level during construction (31). Henschel probably understood his error in the building technique after the collapse and he rebuilt – we think on scaffolding – the courses in the critical zone, reinforcing them by filling the tubes with mortar (32).

Back to the hanging model reconstruction which shows one important aspect not yet discussed. We may expect – without proof however – that Henschel had developed his hanging model method after having read about the famous hanging model by Giovanni Poleni (1748), which was made in order to analyze cracks in the hundred year old cupola of St-Peter's in Rome (33). Both hanging models describe a cupola via a two dimensional model, with simulated weights of a cupola sector. However, an important difference between the hanging model by Poleni and the one we reconstructed by Henschel is that the first only described forces in vertical direction, along the meridians of the cupola, whereas the latter also considered ring-compression forces in horizontal direction. As one can imagine, any structure, including one of optimized shape, bears its load in several directions or, in other words, bi-axial compressions occur in any part of the cupola. If the hanging model is made like the one by Poleni, only the meridian forces, which are mono-axial, are considered and thus the strength of the cupola material is not used to the optimum. The shape of such a hanging model without referring to ring-compression forces is only optimized to a certain degree. Henschel managed to include the ring-compression forces into his form finding model and thus surpassed, from an engineering point of view, not only his contemporary colleagues but also the three-dimensional hanging model by Gaudí (34).

Henschel's success is greater than his misfortune with the hanging model method. The extreme lightweight cupola, notwithstanding the negative conditions of smoke and changing temperatures which are part of the forging process, now stands over 160 years without the necessity of structural improvements. Few other cupolas, such as the flat tile vaults originating from the Catalan tradition and built in the USA by the Guastavino Company, will match the thinness of Henschel's forge. The radical rationalism of its designer, with the methodology of an inventor of machines, is similar to Gaudí's concise structural approach.

Gaudí's use of the catenary

The catenary, used as a method to define an optimized arch shape, was only used in a few cases by Gaudí. His favorite arch shape was the parabola in various proportions, or at least shapes which are very much like the parabola and not catenaries (35). The relationship of a parabola and a catenary from a statical viewpoint is defined. A parabola is an optimized arch form for a constant load distributed along the horizontal plane and the catenary is an optimized arch form for a constant load distribution along the chain's shape. Strictly speaking, none of these load distribution descriptions fit with Gaudí's built structures. In most cases the arched structures figure as window or door openings in a more or less continuous wall, often as diaphragm arches. However, Gaudí often used the parabola shape in a cantilevering type of structure which, as Jan Molema pointed out, possibly stems from similar type of structures in the Maya culture. Arch-like openings in cantilevered masonry structure can be optimized to a certain degree and then show a curve similar to parabolas or catenaries (36).

All this shows a certain ambiguity between static theory and practical solutions. Static theory concluded that the parabola for one structural type and the catenary for an other are "optimal" shapes, but in Gaudí's design praxis the border conditions mostly differed from these specific structural types. We may conclude that openings in wall structures can be made with many different shapes, as Western and Arab architectural history demonstrate.

Gaudí was well aware of this and in two cases he shows us the relativity of the perfect arch shape.

In the first example, we recognize the architect´s desire to play with structural solutions. In Bodegas Güell (37) in Garraf, designed in 1895-1901 together with his collaborator Berenguer, the gateway building shows a parabola arch in masonry. On the inside, this gateway can be closed by a chain net, suspended from a steel post frame. The chain net can be closed by a special device which pulls the net's side border at regular levels into a locked position on the wall. The interesting part, from a statical viewpoint, is that in this particular case the upper border curve of the net is very close to the exact parabola shape. If one observes the way the chain net is made, one can recognize a rectangular net grid, which is suspended from the curved upper border with the help of vertical chains. Thus, the chain net can be regarded as a statical demonstration model of the parabola arch behind it (38).

The second catenary example shows Gaudí's pragmatic attitude towards the building praxis, which he gave similar care as the statical aspect. In the literature, the *desván* structure of Casa Milá is described as a continuous row of parabolic diaphragm arches, which, parallel to the varying spans on the top floor, increase or diminish in height. The famous stepped shape roof landscape was thus generated by the diaphragm system underneath. The parabolic shape was long taken for granted and Bergos drew such an arch with a parabola intrados (39). In an interview of Bassegoda Nonell with José Bayó Font, the builder in charge of Casa Milá, Bayó confirmed that it was decided during the building process to make these arches after the true catenary shape (40). The reason was not primary statical, but mainly to achieve an easy way to generate the many different shapes in full scale for the carpenters to make centerings. On a long and high enough wall, a chain, suspended from two nails in varying positions, served for this purpose and the carpenter could trace the shape directly from the drawn catenary image on the wall.

Again, it should be stressed that these descriptions of the use of the chain, pulled by weights into the parabola shape, as in Garraf, or the true catenary in the Casa Milá desván, show a superior insight in statics and should not be misinterpreted as a confused or arbitrary use of that particular shape.

Gaudí's first hanging model (41)

Gaudí's initial acquaintance with the hanging model idea has not been dated up to now, but it was – like stated before in this contribution – a fairly widely known part of 19th C static theory throughout Europe. Gaudí would therefore already have heard about the theory during his architectural studies in Barcelona. On the other hand we know his first hanging model fairly well, documented by only one photograph, which, was published, inverted, by Ràfols in 1929 and described as a hanging model for the construction of a church (42). The model can possibly be dated around 1898 and figured as an important step in developing Gaudí's hanging model method, preparing the hanging model for the Colonia Güell church. The reason that this small photographic source is sufficient for a plausible analysis of the small hanging model, is on account of Gaudí's clear and synthetic attitude.

Let us start with the synthetic aspect. The model, consisting of white strings, weighted by chains of various lengths, is photographed against a neutral background, thus showing the design shape fairly clearly. The contour of a man figure enables us to estimate the dimensions, which suggest a modest sized church. Another important feature is that only half the design was executed in the hanging model, suggesting a mirror-symmetry on the longitudinal axis.The tops of five domes are held in position by vertical, tightly stretched strings in such way that at any string meeting the symmetry plane passes only horizontal vectors. The main pillars and arches are repeated on the empty side of the model, surely in order to enhance the effect of a complete church. So, without yet discussing the statical impact of the model, we can observe that Gaudí visualised in this hanging model the conclusion of both the architectural and structural shape at the same time. The taking of a decent photograph of the hanging model can be regarded as part of the synthetic design process with the hanging model, in which the architect's role is reduced to rather few opportunities to influence the form-finding process. The architect can keenly observe the final form of the hanging model during the final stages, contemplating the way to translate this optimized structural shape – in inverted position – into a building.

The interpretation of the distribution of the model strings and of the architectural design may be analyzed from the adjacent illustrations. We see a centralized church, a high dome which is surrounded by ten other domes and with an exterior wall zone with integrated balconies on three levels. Some aspects show that Gaudí attempted to keep his design in rather conventional architectural forms. Most important in this respect are the eight main pillars under the domes which are vertical up to a certain height. Gaudí obviously disliked the idea of a hanging model shape dictating oblique directions of the elements in a chaotic order. Of course, in order to achieve the vertical axis of the main pillars, Gaudí was forced into a careful analysis of the weight distribution in domes and the wall zone. The other important discovery is the concept of a wall zone in a double-walled or better "cellular" way. By this and by the fact that the external perimeter was conceived as gently curved, following the domed structure, he could achieve a rigid structure with thin and lightweight walls.

The hanging model for the Colonia Güell church design (43)

When Gaudí was asked in 1898 to design a church for the workers colony Colonia Güell, he had reached maturity, in the sense of an architect with a specific interest in masonry structures as early works show: Palacio Episcopal in Astorga, Palau Güell, Colegio

Teresiano, and the project for a Franciscan mission post in Tangiers. In retrospect we recognize at that time a successful architect, at the peak of his creativity towards a new, organic style, void of historisms. During the design and construction period of Colonia Güell Church (1898 -1914) he would add incredible new solutions to the Catalan vault tradition in a series of buildings like Bellesguard, Park Güell, Casa Batlló and Casa Milá. After leaving the unfinished building of a crypt of unique architecture, Gaudí could positively conclude from the hanging model era that he had successfully completed an experiment necessary to finally conceive the Sagrada Familia.

The working team who helped Gaudí was well trained if one compares it to other projects of rather modest level. The architects Francisco Berenguer Mestres, José Canaleta Cuadras and Joan Rubió Bellver were assisted by a mechanical engineer Eduardo Goetz. For colorful decorations, Josep-Marìa Jujol was added to the team during the construction period. Important was also the photographer Vicens Villarrubias, a sculptor by profession, who photographically documented the hanging model during several stages of its development.

A pine tree covered hill was chosen as a building site for Colonia Güell, which was part of Santa Coloma de Cervelló near Barcelona. Some details about the building site are interesting and confirm Gaudí's ecological sentiment. The more or less symmetrical building was put on an axis centred on a specific pine tree. This pine tree was carefully rounded by the border of the stairway leading towards the church level (44). The direction of the church axis is not exactly towards the east, but at a right angle to the slope of the hill. The church base, the only part that was built, consists merely of a crypt, the stairway to the church and some sacristy space. This volume is partly dug into the hill, within a spacious hole with careful drainage and ventilation elements, which avoid the introduction of moisture.

The workshop for the hanging model was a simple structure, at short distance west of the building site. Aspects of this modest workshop are visible on some photographs of the hanging model (45). It is interesting that some of the documentary photographs of the hanging model were taken from a certain distance, which suggests that Gaudí had conceived the workshop with foldable windows, which could open up to rather big dimensions, necessary for both natural light during photographing as well as for a full view of the model. (46).

The hanging model itself was suspended from a plane underneath the workshop‘s roof. The dimensions of the model in a 1:10 length scale (and a 1:10,000 weight scale) are huge and much bigger than the small hanging model, which probably was in a 1:100, or maybe 1:50 length scale. The length of the hanging model was about 6 m and the height 4 m. It was the biggest hanging model ever established. As a rough concept by Gaudí one may conclude that he wanted to make a similar church as the one of the small hanging model, but with a crypt storey. An important new element is the stairway towards the church entrance, which turned out to be an asymmetric element. This meant that Gaudí had to forego a mirrored symmetry and depict the whole building in the hanging model. As we will see, he made ample use of the potential to introduce asymmetric elements, like the interior stairways, and even different numbers of pillars in the side bays, but the overall shape still has an organic kind of symmetry.

Hierarchy in any structure is relevant to add transparency during the design process and to avoid a hybrid non-calculable force behavior. Gaudí conceived the five nave church with a tendency towards centralization, departing from a similar core as the one of the small hanging model (47). This highest hierarchical level consists of pillars, and adjacent pillar branches, which are connected by arches at some levels. The walls are simulated by vertical strings in the hanging model. The domes are made by radial strings and the towers figure as high domes, mostly with the same number of strings as the domes have.

Why was Gaudí's hanging model so big? One of the reasons is that he developed a rather big detail to connect the strings. It was a wood block holding the strings by a screwed on plate. The wood block was shaped octagonally, probably in order to keep it firm during screwing and also to avoid disturbing the free fall of the strings. Also the weights, in a weight-scale of 1:10,000, being linen sachets filled with lead shot, turned out to be wide and their position in a strict vertical direction underneath the suspension point would be in danger if the sachets touched other elements. The making of the small hanging model will already have been difficult enough technically and when Gaudí planned the bigger model, he must have figured that a big scale was easier to build and to document when measuring. Another reason will have been to increase the exactness of the static results (48). Photography inside the model is easier when the model is rather big.

Why did Gaudí and his team need the long period of 1898 till 1908 to build the hanging model? A main reason is that he maximized his idea for this church for a modest workers community, and really struggled with himself and the hanging model to produce the best possible architecture. In the course of this he lost substance, one even could say that the continuation towards a finished design was confused by many architectural ideas, which were unnecessary for the main point, processing the hanging model towards a built structure (49) Whereas the hanging model was finished in such a way that, in our opinion, no theoretical errors exist which would make the completion of the building impossible, the subsequent building phase from 1908 to 1914 had to stop for financial reasons (50). The considerable time of the building phase was, similar to the production of the hanging model, a result of the huge complexity of the masonry details which Gaudí himself had to supervise. As an experiment, however, preparing the final design phase of the Sagrada Familia, the Colonia Güell church and its hanging model must be regarded as a full success.

How did Gaudí find out the weight of the sachets for the hanging model? The mutual influence of shape and weights complicates the processing of a hanging model. The weights have to be calculated on the basis of the specific weight of the material, e.g. the masonry – 2,000 kg/m³ (51) –, the thickness of the planned building element, constantly 30 cm for the church's wall or 45 cm for the crypt and towers (52), and the surface area (height x width) of the building element. Since the hanging model is built as a three-dimensional net of strings, from which the weights are suspended, the weight instantly influences the resulting shape. But the model builder in charge has to decide the weight amount, before he can approve in the model, whether he estimated the dimensions, simulated by weight sachets, correctly. Thus only a very painstaking process could solve this problem and step-by-step the reworked sachets came nearer to a perfect coherence with the given shape.

Luckily some important documents about Gaudí's hanging model were preserved, and they illustrate how Gaudí did calculate the weights. On these drawings (53) individual arches of the stairway are depicted. Such a drawing shows a thrust line of the arch, sketched by a collaborator in an estimated way, probably basing on dimensional data from an earlier model phase. The thrust line springs on both sides from a column or a wall. The top of the arch is a straight line, suggesting a hand rail of the stairway. The bottom line of the arch is represented by a dotted line, sketched at a small distance parallel to the thrust line curve. The length of such an arch was distributed in equal wide parts and the technical drawing contains further a calculation of the weight of these parts. The amount of an individual weight in these drawings reaches up to 2,700 kg. Gaudí himself sketched the architectural form on some of these drawings. The number of sachets for an arch differs from three to about nine, in accordance with the length of the arch and of the curve type, which should keep its organic curve, in order to produce theoretically correct vectors to the connecting building elements (54).

The number of drawings prepared for the hanging model in this way must have been over one thousand or even considerably more when such drawings had to be made repeatedly, as suggested here. The total number of sachets is estimated by us as 5,000.

In the case of the columns, Gaudí reversed the method of determining the dimension of a building element. The column‘s surface area was calculated from the maximum compression strength, being 8 kg/cm² (brick masonry) and 40 kg/cm² (basalt columns) and from the weight the column had to carry (55). The weight on the columns was measured in the model strings by a device called "dynamometer" (56). For the crypt part, all string loads for the columns and wall parts are documented on an original plan (57). In this case Gaudí did not charge the weight of the columns themselves in the loading of the model. The reason for this will be that most columns were deliberately vertical in their heavy part, near the ground. In such cases the considerable weight of the column trunk itself, in comparison to the weights or loading of the buildings elements they carry, does not affect the vertical compression line inside the column trunks, since the verticality results as a sum of vectors from the upper building elements resting on top of the trunk, leading to zero horizontal thrust forces inside these columns (58).

An important discussion in respect to the formfinding capacity of the hanging model is the question whether Gaudí influenced the hanging model shape. Generally spoken, during the reconstruction work with the hanging model, it is stated that Gaudí initially followed the concept of a free hanging string, with catenary-like shapes, without taking into account the building element's weight. The problem of this form finding strategy, which could be called allowing for the "natural" shape, was that some building parts would have been problematic in a functional sense. The most important problem was that the exterior walls tended to be noticeably oblique, which of course was not nice for the church users with such reduced dimensions of the floor's height. Gaudí moderated the problem of the inclined exterior wall by adding weight to the wall‘s top zone, in the shape of battlements, like the pinnacles of gothic architecture. Gaudí found a general and inventive approach with the peripheral position of all towers, which pull the walls outwards adding volume to the church's interior. Important issues in this respect are the horizontal strings, which Gaudí distributed at regular levels connecting the vertical strings for the wall zones. These horizontal strings, paralleled by arches of balconies and domes, can be interpreted as activators of the tower weights in zones without towers (59).

The famous Gaudinian tower shape, built at the Sagrada Familia in its mature shape, has been studied in the hanging model. Here we can see that Gaudí manipulated to a considerable extent the shape of the "natural" raised dome of constant thickness. As one can imagine such a dome shows a section that is rather rounded. Compared to the catenary shape, which correctly shows the shape of an arch or barrel vault of constant thickness, a dome-like tower of constant thickness, simulated by radial strings as in Gaudí's hanging model, shows a rather rounded top. One could say that esthetically such a tower is not very pleasing because neither the real top nor the position of the vertical tower axis is visible from ground level. Without affecting the rigid theory of hanging models, Gaudí added a considerable weight on top of the six towers of Colonia Güell church, in the shape of decorations, thus correcting the towers‘ shape until the characteristic steep proportions were reached (60).

Another important aspect of the towers as part of the general structure is their increasing influence on the total weight of the church. In general, weight is saved to a great extent because of the static optimizing ability of hanging models. Thus the gravity problem may be solved, but in reality not only self-weight gravity produces forces, but also dynamic loads, like visitors or the bench layout, and also wind load. As a result Gaudí used the towers to increase the weight and the force level inside the elements to a certain level in order to avoid that wind load which would cause tension forces in the building parts.

In order to adapt to changing moving loads, Gaudí may have added a very interesting reinforcing aspect to his hanging model. The structural part, which may be loaded by the main moving load, is the church floor. Thus in the crypt, the true compression line inside diaphragm ribs, pillars and walls were possibly critically different from the hanging model, changed by the effect of moving load, let's say during a procession, which went from left to right. The most important problem could possibly occur on the exterior, were only few vectors mix, producing the resultant force in the crypt walls. Gaudí built these parts, the walls of the crypt, with strings, alternating between an inclined and a vertical position. This may be interpreted in such way, that in such cases when the resultant vectors tend to depart from the church floor structure, their angle changes and either the vertical or the inclined wall part bears additional forces. So, where normally the hanging model responds to any load change by changing its shape, in this case, the fixed position of the crypt's wall, modelled by both inclined and vertical strings, avoids this effect. The load level in the strings will change to a certain, but still safe, degree.

The documentation of the design phases during the construction of the hanging model is an important issue. As we have seen in earlier cases, the question of measuring a catenary and – from our view point simply – reproducing it at a larger scale was probably the most important barrier for architects in the 18th and 19th C. to designing with the chain model. The reproduction of "free" forms, like hanging models is even more problematic (61).

Especially the documentation of the design phases was a problem. In ordinary design praxis, the architect fixes the design in drawings before building a physical model. Both the drawings and eventually extra models, are documents of the predesign. In Gaudí's design with the hanging model the design phases disappeared during the progression of the hanging model.

Gaudí documented the design phases by photographs. These turn out to be of excellent quality, and they must be interpreted as an important step in scientific photography. All still existing photographs are taken from seven view points, thus showing the entire model (62). If changes in the model were needed, photographs of earlier phases could be taken into account for comparison. The quality of the photographs is partly due to the care with which the model was lit, using white sheets as background. Another interesting way to add some volume to the completely transparent net of strings was the application of paper sheets in the wall regions.

In the end, the most important aim of the photographs became their use as a background for over-painting. Gaudí simply inverted the photographs of both the interior and exterior and used them as an image of the church to be designed, adding his visionary architecture by painting windows, decorations, masonry a.s.o. In the interior Gaudí added small discs to the column strings , with the same radius as the column. These discs gave, on photographs from any view point, the true dimension of the columns, enabling the drawer to sketch the columns' shape in correct perspective shortening.

The other documentary step was the measurement of the hanging model, equal to some 10,000 points in space. Again Gaudí was eager to make an exact as possible result. He placed metal plates of reduced dimensions beneath the hanging model in a horizontal fixed position. With a burin he etched the projected position of the model points in the metal plate, adding in the height level and some kind of coding. These metal plates were directly used on the building site in order to fix the interior and exterior surface of the walls or positioning the pillars in the correct inclined shape, held by scaffolding (63).

Gaudí's use of graphic statics

A short and incomplete listing of Gaudí's designs with graphic statics is relevant here, since the method of hanging model and the graphic static calculation show, in the case of two-dimensional structures normally the same result. Both can state the static behavior of a given structure (64). But the three-dimensional hanging model has as a superior feature, the possibility of form finding an optimized stone structure.

Because of the big loss of Gaudí drawings, only little evidence of graphic static calculation remained. Gaudí's general interest in modern statics is proven by the story about the iron water tanks for the fountains of parque de Ciudadela (1874), designed by Fontseré, for which as a student Gaudí delivered a full calculation. This static proof was such that his professor counted it as a full exam for which Gaudí was then exempted (65). Early use of the graphic static method by Gaudí is stated by correspondence about the Palacio Episcopal in Astorga, dating from 1887(66). The famous drawing of a viaduct in Park Güell, was probably made for an exhibition in Paris in 1910 (67). The drawing takes into account two kinds of loads, not only the gravity load of the viaducts' upper road, but also the thrust forces from the hill beside.

An interesting drawing for the Sagrada Familia is the one depicting a predesign of the passion façade (68). The structure, a three bay arch, is solved in an aequilibrium of the horizontal thrust departing from the different high arches. Thus the resultant forces of the two pillars on both sides of the central axis have a vertical direction. As in the case of the hanging models, the vertical positioning of the pillars show Gaudí's talent for structural proportioning, rather than proof of a conventional design.

The problem Gaudí always deals with – surpassing conventional architecture – is his urge for lightweight structures which need a perfect shape. In the case of the Sagrada Familia church, Gaudí designed many proposals and probably all were proofed in graphic static calculation. Before discussing these, it is interesting to know, that Gaudí, with Rubió Bellver and Jujol as collaborators, worked on a restoration of Palma Cathedral, one of the bravest gothic structures and of similar dimensions as Sagrada Familia church. Rubió Bellver, being one of the few older collaborators of Gaudí who wrote on structural subjects, has published and commented a static analysis of Palma cathedral including a graphic static calculation (69). Although Rubió does not mention Gaudí in the publication, it may be clear enough from the restoration work that Gaudí was acquainted with this analysis and even had contributed to its interpretation. On the other hand it is typical for Gaudí's secure methodology that for the Sagrada Familia designs he made comparisons with older structures, like Palma cathedral (70).

The statics of the last Sagrada Familia design is published by Sugrañes (71). The half-section of five nave Sagrada Familia shows many features like the Colonia Güell church design. The inclined and branched columns are of course specially of interest, but also the use of ruled surfaces, which in an experimental way were executed as vaults of the exterior stairway and wall parts of Colonia Güell church. The degree of inclination of the pillars is reduced by an intelligent weight distribution, among this a pyramidal building element on the side bay's roof (72). Reducing the columns' inclination is positive for the building phase, since it avoids scaffolding, as the column trunk stands on its own, the biggest in the crossing measuring about 22 m height. An important difference from the concept of the Colonia Güell church and the Sagrada Familia is the way that Gaudí interpreted the branched column (73). In the Colonia Güell church the column and its branches are part of an equilibrated system of mostly linear elements, a skeleton, shaped by the strings of the hanging model (74). In Sagrada Familia we find the structural concept of trees in the branched columns, which individually may stand safe due to small tension elements in the vaulted zones and roofs.

An often discussed question is why Gaudí did not use the hanging model method in the Sagrada Familia. The reasons why he relied on the pragmatic graphic static method, may be found in both the design and building process and in statics itself. Planning the Sagrada Familia turned out to be one of the most complex and long term design processes of its time, Gaudí working between 1884 till 1926. Any hanging model – as is stated by modern use of technical models – due to its delicate details, tends to be exact during only a relative short period. On the other hand statically the Sagrada Familia was being built from the start in vertical sections, which is not coherent with the synthetic aequilibrium of hanging model based structures (75).

Gaudinism and static calculation

As interesting as Gaudi's predecessors are in understanding how in history optimization was looked for in structural design, a universal effort of mankind, as intriguing is the question how Gaudi's followers acted with his experience. Did all knowledge simply disappear, like it seems was the case with Hübsch and Henschel, or did there exist some kind of specific influence, what one could call Gaudinism?

The latter is the case which I hope to demonstrate by some examples. My first example of Gaudinism is a theoretical work, contemporary during Gaudí's lifetime. One of Gaudí's collaborators, Joan Rubió Bellver wrote an article „Difficultats per arribar a la síntessis arquitectònica" on Gaudí's design method with the hanging model (76). As an architect Rubió Bellver designed secure Neo-Gothic churches, in which he used graphic static analysis.at least in one case The very humble church for the Monestir de Sta. Familia in Manacor, Mallorca, shows a surviving design drawing on mm-paper including in a section a graphic static calculation. The church, dated 1906-1908, shows a wooden roof on equilibrated diaphragm arches in limestone (77). I suggest that graphic static analysis was also used to optimize the design of the bigger parish church in Son Servera, Mallorca, (1910) by Rubió Bellver, which remained unfinished (78).

The architect Josep-María Jujol, brilliant colorist in the work of Gaudí, displayed in two church designs a mature interest in statics, whereas in other work his creativity and sentiment drove him to an abundant decorative architecture, with sometimes witty or even bizarre results. The church in Vistabella, 1917-1923, shows a crossing with diaphragm arches of parabolic intrados shape. From the fact that these arches carry a tower with buttressing elements, following the shape of a tangent to the parabolic shape, one may guess his sincere aim to use the parabolic arches in a rational way (79).

A further development of this structure focussed design approach was the church "Santuari de Montserrat" in Montferri, 1925 - approx.1930, like Vistabella a small village near Tarragona. The design for Santuari de Montferri ended in an unfinished work around 1930 and the church was completed 1989-1999 by Bassegoda, Jané, Jané, Tomlow (80). Montferri church has all arches executed in a catenary shape, thus referring to Gaudí's hanging model in a simplified interpretation.

Also later generations, were fascinated by Gaudí's hanging model, as we can see from the finishing project of Montferri church. Like his father, Bonet Garí, and Puig Boada, the present architect of the Sagrada Familia, Jordi Bonet Armengol experimented with ruled surfaces like the hyperbolic paraboloid shape. For the design of Sant Medir church in Barcelona, 1955, Bonet developed a small and creative hanging model, which allowed him to balance in one section a series of hyperbolic paraboloid shells (81).

Conclusions

This summary of Gaudí's use of calculation methods in optimized vault design, amidst earlier and later applications of catenary, hanging model and graphic statics, provides a topic which for most architects and admirers of architecture goes beyond their usual interests. I see three aspects causing the strangeness of the catenary theme connected to Gaudí.

First aspect: A caricature which figured Gaudí as an idiosyncratic architect who had developed a mysterious machine called "hanging model", and who produced archaic stone labyrinths via an obscure methodology, could stay popular over a long time. The poor documentation of illustration material about Gaudí's architecture, is due to heavy losses during the Civil War 1936-1939. Documents of catholic culture, including architectural designs by Gaudí, were then destroyed by the anti-clerical ruling powers. Research results on Gaudí's hanging model since 1975 made his methods transparent and clarified the question of his professionalism in structural design.

Second aspect: The catenary line, in its historical interpretation, is thought to be a strange element in European architecture. Compared with Euclid's "good old" straight line geometry, the catenary is judged on aesthetic grounds as primitive, crude, archaic. The increasing relevance of ecology for contempory culture may help to integrate the catenary and other "Natural Structures" in our architecture. This wish is the continuation of a specific philosophy within architectural theory from the Romantic period of the early 19th C.

Third aspect: The catenary and the hanging model used for formfinding in structural design, asks from architects and engineers – and investors – a radical departure of their usual attitude to the leader's role. The captain and the compass on the design ship is not the architect but the synthetic nature of the formfinding process. The resulting shape has to be contemplated rather than directed. Deliberate changes by the designer deal with parameters and conditions of the result, rather than with the shape itself. Such an architect's office is possible and intriguing modern examples exist.

Notes

1. Complex geometry forms in the work of Frei Otto (soap bubbles) and in the work of Félix Candela (ruled surfaces; especially hyperbolic paraboloid), are chosen in structural design for reasons related to their statical impact. In that sense one can select a sphere as a logical form for a pneumatic construction, like air-halls. Candela's calculation method of reinforced concrete shells makes use of the simplicity of the straight line in a hyperbolic paraboloid shell. See also Tomlow Geometry 1996.
2. Compare to this theme Burry 1993, Bonet 2000, Tomlow model 1989, Tomlow Gaudí 1996.
3. Compare for the history of hanging models Graefe 1986, Bassegoda 1986, Kurrer 1997, Molema 1992, Tomlow 1993, Tomlow, Trautz 1994, Huerta 1996, Trautz 1996.
4. Please allow me a philosophical annotation. One could remark that people who are seriously involved with hanging models, naturally become members of an esoteric club. The traditional notion of Gaudí as a genius, whom none of his pupils or followers could intellectually or creatively match, may in this sense be replaced by an equal position of Gaudí amongst gaudinists, all working towards a common goal.
5. About the use of the catenary by Wilhelm Tappe see Germann 1974, Graefe 1986, Tappe 1818. Rainer Graefe has kindly offered me additional information on Tappe.
6. His low training level in writing is shown by numerous errors in quoting names, including writing the same name in different ways (Seb. von Maillords / S.V. Mailard). Also his description of quoted publications is incomplete.
7. A peculiar use of his architecture is monument designs in honor of the nobility like Friedrich der Große of Prussia.
8. More than the eight volumes were planned, see Tappe 1818, 8. Heft, p.36.
9. His arguments were: cheap building material of sun-burned mud bricks, simple straw roofs fixed into the vault masonry joints, minimal surface of the ground plan, small window surface, a pleasing rustical exterior which fits well in the rural culture of Germany, simple building technology, wood hardly necessary, fire proof building system. Of course, because of the straw-covered vault, critics argued that this would not be fire proof, but on the contrary very dangerous. Tappe was very serious in his response and with some plausibility he argued that tile covered roofs were too expensive and not very suitable for a cupolar shape, and that in any case the straw would quickly burn on the outside of the house only. .

10. Tappe demonstrated two ways to generate an ellipse, one as an elongated circle and the second with the well known method of a rope fixed to two poles in the center points, Tappe 1818, 6. Heft, fig. 83, 84.
11. See Tappe 1818, 6. Heft, p.19, fig. 90-93. Theoretically any chain under normal gravity loading will show an inclined springing point. See also Graefe 1986 who described the error in his text.
12. Sadly one has to conclude that up to now only few hanging models built by scholars and students throughout the world are theoretically, and as formfinding models, correct. A source of the model technique and other aspects are the publications of the Stuttgart Institute for lightweight structures (IL), formerly led by Frei Otto, see f.i. Gaß 1991.
13. Comparing with Tappe's form vocabulary especially interesting is a chase hut in the Pyrenees, chalet refugio de Catllaràs, Pobla de Lillet, 1904-1905, which is contributed to Gaudí. Bassegoda 1989, p. 469.
14. The hanging model method by Hübsch is dealt with in Germann 1974, Graefe 1986, Tomlow 1993, Tomlow Geometry 1996 and in detail in Graefe 1983.
15. „In welchem Style sollen wir bauen?", Karlsruhe 1828; title translation: "In which style we should build?"
16. The buttresses have a stepped exterior surface, thus demonstrating the inclined thrust line of the hanging model method.
17. Hübsch claimed the development of the hanging model method as his own invention. In my opinion he is correct in this, since the much earlier discovery of the physic principle „reversion of the catenary is an optimized arch shape" as such does not lead automatically to a successful design and building. His hanging model method can be considered as original.
18. It would be interesting to compare German philosophy of the Romantic period with English or French philosophy on the term „natural structures" and the impact of the findings of physics on the architectural production. See Tomlow Geometry 1996, p. 75 for quotations by Hübsch and Henschel about „natural structures".
19. Compare Puig 1981, quotation Nr. 11, 12, 145, 178, 187.
20. Tomlow 1993, Tomlow Geometry 1996, Trautz 1996. A reproduction of the reconstructed hanging model can be seen in the presentation of the Henschel firm, Hessische Landesmuseum, Kassel.
21. The Henschel firm suffered an almost complete loss of its factories through bombing during World War II. Coincidentally the Henschel foundry survived and now serves as an auditorium for the Gesamthochschule Kassel.
22. For biographical information on Henschel, see von Mackensen 1984.
23. Some quotations on the term natural structures and with side references to his hanging model method in Tomlow Geometry 1996, p. 75.
24. The interesting architectural choice for the Roman Pantheon is discussed in Sander, H.: Der Kasseler Kuppelbau, ehemals Gießhaus der Lokomotivbaufirma Henschel, in: Zeitschrift des Vereins für hessische Geschichte und Landeskunde, Band 90, 1984/1985, p. 252.
25. The rediscovery of Henschel's hanging model departs from a short mention of hanging models by Henschel and Hübsch in Hagen 1862, p. 24f. Our complex reconstruction of the hanging model and its interpretation are described in Tomlow 1993. Students Fässler, Moser, Trautz-Fülöp assisted in the model work and and Tilman Hörsch prepared a civil engineering diploma thesis with additional calculations on computer basis, see Hörsch 1993.
26. The reconstruction model is two-dimensional and shows only one half, which is theoretically sufficient. It has a length scale of 1 inch (approx. 2.4 cm) = 1 mm, with a total length of some 40 cm. The load scale is 1 cm chain = 100 N. The chain type is gilded tombak A40/150 and steel rings. In preparation of the model building the following steps have been done: calculation of the weight distribution in the design drawing, concerning the fact that in a cupolar shape the elements to be calculated grow narrower towards the top. Calculation of the gravity points for the loading of the hanging model with chains. After montage of this hanging model the shape matched within 1 mm the prepared drawing of the circle segment.
27. It is important to point out that along the cupolas footing, an iron tie-ring takes the thrust forces from the cupola, thus loading the walls only with vertical compression forces.
28. See Tomlow model 1989, p. 136

29. The cupola was later covered with bitumen and after the restoration of 1977 a wooden protection roof covered by sheet metal was added, increasing the cupola thickness by some 25 cm.
30. The collapse was reported by visitors; compare Tomlow 1993, p. 153.
31. The critical level was calculated with the help of the hanging model method for different stages of the building process. The critical level of the ring-compression turned out to be 10 to 25 N/cm². This rather low force level was probably fatal because of still weak mortar in the final course during construction. See Tomlow 1993, p. 169f, with also a short demonstration of the Hörsch findings by computer.
32. Another change in the final shape is that he simplified the situation by reducing the amount of steps in the cupola thickness. Contradicting to what one may expect, the final shape is less coherent in static terms than the first design, as we could prove with both the hanging model and the finite element method. See Tomlow 1993, p. 161.
33. The beautiful illustrated original publication of this analysis is: Poleni, G.: Memorie istoriche della gran cupola del tempio Vaticano e de' dani di essa, e de' ristoramenti loro, Padua, 1748. See also Graefe 1986, Benvenuto, E.: An introduction to the History of Structural Mechanics, New York 1991, Part II, p. 358, Straub: Die Geschichte der Bauingenieurkunst, 1992, p. 191ff. The hanging model by Poleni is not considered further in this contribution, since it was not used by him as a formfinding model, but mainly as a means of statical analysis.
34. The way to consider in a two-dimensional hanging model ring-compression forces was developed at the Institute of Lightweight Structures, Stuttgart University, under the direction of Frei Otto, compare Mack, R.: Modellstudien zum Tragverhalten der Kuppel des Pantheons, in: Zur Geschichte des Konstruierens, ed. by R. Graefe, Stuttgart 1989 p. 38-43, Tomlow 1993. In Tomlow 1997 is published a hanging model analysis of Küçük Ayasofia Mosque, formerly a byzantine church of 527-536 A.D. on request of Prof. Görün Özsen of Yildiz University. In the modeling assisted Metschl and Freudenberg of Hochschule Zittau/Görlitz FH.
35. The use of parabola shape for arches and windows by Gaudí is documented by Hiroya Tanaka in: Tanaka 1991, p.8-9.
36. Compare Molema 1992. In the IL Stuttgart these cantilevering type structures were analyzed, compare Gaß 1991, p. 4.12-4.15.
37. Compare Molema 1992, p. 199-211.
38. I am grateful to Manfred Speidel who informed me of this interpretation of the Bodega Güell gateway and chain fence.
39. Compare Bergós 1974, p. 101
40. Compare Bassegoda 1986, p. 553, and Roca 1996, p. 56
41. The interpretation of this hanging model was elaborated in my doctoral thesis at the IL, Tomlow Model 1989 p. 222-227. Tomlow Madrid 1989. The research work was encouraged by the favorable conditions of the IL, around 1984, which showed all about model techniques, much alike the ones described here.
42. Ràfols 1929, fig. p. 200, text p. 144/145. Kind information by Jan Molema. See also Tomlow model 1989 p. 262 for a discussion whether this model could have served for the non-built chapel design for Park Güell. The model was kept in a cupboard in the Sagrada Familia workshop, as Ràfols states. Please note that the small hanging model was not made as a design for the Sagrada Familia, as erroneously has been suggested.
43. This chapter is based on Tomlow model 1989 with a literature list on p. 266f for its sources, compare also Tomlow Madrid 1989. The doctoral thesis departed from the reconstruction of Gaudí's hanging model (1983) by Rainer Graefe, Frei Otto, Jos Tomlow, Arnold Walz with team, a commission by Harald Szeemann of Kunsthaus Zürich. The reconstruction model, in scale 1:15 was made in Stuttgart University in the Institute for lightweight Structures (IL), directed by Frei Otto. The Gaudí research group Delft, founded by Jan Molema and students, was engaged in the investigative and executive work. Joan Bassegoda Nonell, Yasuo Matsukura (Matsukura 1978) and Isidre Puig Boada (Puig 1976) helped with information and copied sources. All participators to the reconstruction work added with profusely discussions to the new insights in Gaudí's working method on the

hanging model, until then obscured by missing evidence because of destroyed documents. The reconstructed hanging model is now part of the Sagrada Familia Museum. Since 1989 many research result add information about the Colonia Güell church project. Prof. Rainer Graefe, co-author of the reconstruction model, renews since 1997 in a scientific project by Innsbruck University of considerable extent and creativity, efforts to reconstruct the Colonia Güell church design by Gaudí, compare Graefe 1999. The first publication of the Gaudí research group, Gaudí-groep 1979, contains in my contribution on Colonia Güell, p. 168f, an interpretation of Gaudí's hanging model as a chain model, built by Michielse and van Veen on the base of Puig 1976.
44. The pine tree, visible on old photographs was later removed and substituted by a younger tree.
45. The location of the workshop for the hanging model, documented by some remaining parts, was discovered during a restoration in 1999. My reconstruction of the workshop situation of Tomlow model 1989, p. 114 shows an error. The real place of the workshop is some meters north.
46. Gómez Serrano 1996 gives interesting insights in Gaudí's workshop next to the Sagrada Familia, with again special devices for photography.
47. Tokutoshi Torii has analyzed shape comparisons between Gaudí's projects of Tangiers, and the two hanging model churches, compare Torii 1996.
48. Gaudí often worked at a large scale. For instance he prepared gypsum models in scale 1:25 and 1:10 and working drawings for the big tower of the Sagrada Familia were also in scale 1:25 what forced Quintana to draw on the floor because of its huge dimensions. See Bonet 2000, p. 52f. Especially in drawings representing organic or complex geometry shapes, drawing dimensions have to be big enough, in order to give exact information.
49. A good example of the maximizing attitude of Gaudí is a certain pillar, numbered B, which branches in four trunks, continuing in the church level as four pillars, only to merge again to one pillar in the top storey. Tomlow model 1989 p. 153. In the symmetricallypositioned pillar number b only one continuous vertical element exists, suggesting that this structural play is unnecessary.
50. Compare Tomlow model 1989 p. 19.
51. Compare Tomlow model 1989 p. 141.
52. Compare Tomlow model 1989 p. 144f.
53. Compare Tomlow model 1989 p. 141.
54. In the drawings discussed here, no weight of the stairs or the vaults that are underneath appear and in the hanging model only the arches are represented by threads. To what extent a simulation of specific building elements into a hanging model or a graphic static drawing is a constant theoretical issue of any statical calculation method.
55. Compare Tomlow model 1989, p. 145.
56. The dynamometer was developed much earlier and described by in L'Art de Bâtir of Rondelet, compare Rondelet, Tome 3, p. 22, Pl. LXVIII, fig. 4.
57. Compare Tomlow model 1989, p. 139.
58. Also in this case Gaudí did not act methodically repetitive. The oblique basalt columns in the crypt carry the vertical pillar trunks in the church. The reason of the basalt column inclination is the bigger width of the central bay compared to the side nave's width. As part of the ongoing restoration of Colonia Güell crypt, Albert Casals Balgué, José Luis González Moreno Navarro and Pere Roca Fabregat have controlled the inclination of these pillars with graphic statics and the finite element method, and interpreted as not correctly simulated by the hanging model, compare Casals 1993 and Roca 1996, p. 57-59. About an interpretation of the missing church volume in the building, which of course produces a changed course of thrust lines, compare these texts with Tomlow model 1989, p. 170.
59. Compare Tomlow model 1989, p. 165f.
60. Compare Tomlow model 1989, p. 158, 160, Tomlow Gaudí 1996.
61. For the same reason I cannot believe that in gothic times any master builder could handle the measuring and translation of a hanging model, as suggested by some scholars. Compare Tomlow Geometry 1996.

62. All photographs of the hanging model and of the so called over paintings are published in Tomlow model 1989, p. 64-67. Two over paintings have been photographed in its original state, around 1910, and in its present bleached state.
63. For a detailed discussion of the translation of the hanging model into the building, compare Tomlow model 1989, p.152-221.
64. Compare Tomlow 1997.
65. Compare Martinell 1975, p. 205
66. Compare Martinell1975, p. 467f.
67. Compare Collins 1983, plate 44 B and 44 C.
68. Compare Torii 1983, tomo II, fig. 550. Torii dates the design drwaing 1891-1893 and the graphic statics 1901-1906.
69. Compare Rubió 1912.
70. Gaudí's workshop always was more than only an architect's office. Despite his anti-institutional attitude, illustrated by lack of contacts to university circles, his working method included self-financed research in numerous fields. In statics he carried out load-experiments on different materials and obviously as a routine, compare Bergós Masso, J.: Materiales y Elementos de Construcción – Estudio Experimental, Barcelona 1953.
71. Compare Sugrañes 1923.
72. Compare Tomlow Gaudí 1996.
73. For detailed statements about Gaudí's thoughts concerning these questions reported by his collaborators and gaudinist friends Joan Bergós, Cèsar Martinell and Isidre Puig Boada, compare Puig 1981, Martinell 1975, Matsukura 1984 .
74. Please note that the many windows in Colonia Güell church design have as a statical consequence, that the forces really follow the course of the hanging model. If big wall areas are not subdivided by windows and other openings, compression forces could rearrange their course and cause dangerous compression concentrations in areas with too little material or even cause tension forces. Compare Tomlow model 1989, p. 168f. We can observe a similar attitude towards window distribution in the structurally brilliant Mosque architecture by Sinan from 16th C. Ottoman period.
75. Compare also Tomlow model 1989, p. 230, Tomlow Gaudí 1996. The only part where to my opinion a hanging model approach in the Sagrada Familia planning process would be interesting would be the central tower, symbolizing Christ. On the other hand the wind effects on the 170 m high central tower should be taken in account by a statical calculation.
76. Compare Rubió 1913.
77. The design drawing of Monestir de Sta. Familia in Manacor, Mallorca with graphic static calculation, was kindly copied for me by Ignasi Solà-Morales Rubió, from his Rubió archive, compare Tomlow Rubió 1995, p. 283f .
78. A drawn reconstruction of Son Servera church design together with a graphic static calculation of the design, was published in Tomlow Rubió 1995. The reconstruction work was executed by the author with Stuttgart students Reuter, Trautz-Fülöp and Neuss.
79. Compare Tomlow Jujol 1995, p. 67-73.
80. Compare my annotated reconstruction and building report in Tomlow Jujol 1995. About Gaudí and Jujol compare Flores 1982
81. Compare Tomlow 1992, p. 79.

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Illustrations (please note: in publication less Illustrations)
Extra 1. The inversion of the catenary – the chain’s shape – may be used as a formfinding metod for an optimised arch shape. This principle of static theory was discovered around 1700. (Source: Drawing Jos Tomlow)
T1. Wilhelm Tappe. Fig. 16/17 from Tappe’s publication. To the left a three room house with attic and to the right a garden cabin with wooden framed top light. Fig. 16 was built in 1818 as an experimental house. (Source: Tappe, W.: Darstellung einer neuen äußerst wenig Holz erfordernden und höchsfeuersichern Bauart, 1. Heft. Essen, Duisburg 1818)
T2. Wilhelm Tappe. Figures 1-12 from Tappe’s publication with technical information on building his new house type. Fig 9, shows the formwork, held by a central post, which defines the exterior cupolar shape. The cupola could be executed without scaffolding, making use of ring-compression which is established in every fresh masonry layer. (Source: Tappe, W.: Darstellung einer neuen äußerst wenig Holz erfordernden und höchsfeuersichern Bauart, 1. Heft. Essen, Duisburg 1818)
T3. Wilhelm Tappe. A village, combining several types of Tappe’s designs. (Source: Tappe, W.: Darstellung einer neuen äußerst wenig Holz erfordernden und höchsfeuersichern Bauart, 4. Heft. Essen 1821)

T4. Wilhelm Tappe. Fig. 90.-93., Plate 13 from Tappe's publication demonstrates his - erratic – assumption, that the catenary shape of a chain like in Fig. 90 can be influenced by weights into an ellipse, a pointed arch, or a half-circle, all with vertical tangents in the foot points. Tappe preferred of these shapes the ellipse and the pointed arch as sections of his vaulted house type. (Source: Tappe, W.: Darstellung einer neuen äußerst wenig Holz erfordernden und höchsfeuersichern Bauart, 5. Heft. Essen 1821)
T5. Wilhelm Tappe. Designs for gateways, monumental arches, a bridge or an aqueduct, galeries. The bridge design with elliptical arches shows a similar solution as Gaudí's Park Güell viaducts and the bridge project over Torrente de Pomeret. (Source: Tappe, W.: Darstellung einer neuen äußerst wenig Holz erfordernden und höchsfeuersichern Bauart, 8. Heft. Essen 1823)
T6. Wilhelm Tappe. Designs for towers, gateways, a bridge. (Source: Tappe, W.: Darstellung einer neuen äußerst wenig Holz erfordernden und höchsfeuersichern Bauart, 8. Heft. Essen 1823)
T7. Wilhelm Tappe. Monument design for Friedrich der Große of Prussia. (Source: Tappe, W.: Darstellung einer neuen äußerst wenig Holz erfordernden und höchsfeuersichern Bauart, 8. Heft. Essen 1823)
T8. Similar to Tappe's form vocabulary is a chase hut (hunters lodge?) in the Pyrenees, chalet refugio de Catllaràs, Pobla de Lillet, 1904-1905, which is contributed to Gaudí. (Source: Cátedra Gaudí, Bassegoda 1989, p. 469).
Hü1. Heinrich Hübsch. Section of the catholic church of Bulach, near Karlsruhe (1828-1837), with in the right half the compression line as determined by the hanging model method. (Source: Hübsch, H.: Bau-Werke, Carlsruhe, 1. Folge 1838. Courtesy of Landesbibliothek Baden-Württemberg, Stuttgart)
Hü2. Heinrich Hübsch. Test building for Bulach church (1834/1835). (Source: Hübsch, H.: Bau-Werke, Carlsruhe, 1. Folge 1838. Courtesy of Landesbibliothek Baden-Württemberg, Stuttgart)
Hü3. Heinrich Hübsch. The Neo-romanesque appearance in the exterior of Bulach Church. (Source: Photo Jos Tomlow)
Hü4. Heinrich Hübsch. Small buttresses of Bulach Church, demonstrating the inclined compression line determined by the hanging model method. (Source: Photo Jos Tomlow)
Hü5. Heinrich Hübsch. Interior of Bulach Church. (Source: Photo Jos Tomlow)
He1. Carl-Anton Henschel. Working drawing, ink colored, of the Henschel foundry (1837), Kassel, designed with a hanging model. (Source: Courtesy of Hessische Landesmuseum, Kassel)
He2. Carl-Anton Henschel. The Henschel foundry (1837) with the Henschel house, to the left. Aquatinta by J.H. Martens. (Source: Denkschrift aus Anlaß des hundertjährigen Bestehens der Maschinen- und Lokomotivfabrik Henschel & Sohn, Cassel 1910)
He3. Carl-Anton Henschel. The Henschel forge. (Source: Courtesy of Hessische Landesmuseum, Kassel)
He4. Carl-Anton Henschel. Detail Henschel foundry cupula, built of tubular bricks. (Source: Photo Jos Tomlow)
He5. Carl-Anton Henschel. Two-dimensional reconstruction of the hanging model for Henschel forgery. Ring-compression forces are represented by three additional chains with a horiziontal connection. The additional chains are of constant length and the amount of thrust force on the model can be corrected by changing the angle of the open end. Reconstruction model by Fässler, Moser, Tomlow, Trautz-Fülöp. (Source: Photo Gabriela Heim, IL Stuttgart)
He6. Carl-Anton Henschel. Representation of a three-dimensional hanging model for Henschel forgery. Dawing by Ralf Höller. (Source: Drawing Ralf Höller, IL Stuttgart)
He7. Carl-Anton Henschel. Bottom: representation of the topology of a threedimensional hanging model, simulating compression forces in radial and ring direction. Only one section of such a threedimensional model was reconstructed. Top: Representation of the cupula. The vector arrows show ring-compression forces. The remaining thrust force on the cupola base, is taken over by an iron tie-ring. (Source: Drawing Ralf Höller, IL Stuttgart)
He8. Carl-Anton Henschel. Interpretation of the device to establish the cupola's shape during construction, without the need of full scaffolding. (Source: Drawing Jos Tomlow)

G1. Bodegas Güell. Gate house of Bodegas Güell in Garraf with chain net, displaying a parabola-like shape in the suspending chain, because of a regular loading by the rectangular chain net underneath.
(Source: Photo Peter Bak, Gaudí research group Delft)
G2. Casa Milà. Drawing of a diaphragm arch in the Casa Milà desván by Joan Bergos, representing the shape as parabolic. In reality the arch shape is a catenary. (Source: Bergós Masso, J.: Gaudí, el hombre y la obra, Barcelona 1974)
G3. Casa Milà. Diaphragm arches in the Casa Milà with shapes generated by a chain. (Source: Photo Jos Tomlow)
G4. Church design. Photograph of the small hanging model (around 1898), published in inverted position. The structure is only shown half and the cupola's tops are fixed to a firm vertical thread. The model consists of threads with chain weights. (Source: Ràfols, J.F., F. Folguera: Gaudí, Barcelona 1929)
G5. Church design. Drawn reconstruction of Gaudí's small hanging model. (Source: Tomlow, J.: Das Modell - Antoni Gaudís Hängemodell und seine Rekonstruktion - Neue Erkenntnisse zum Entwurf für die Kirche der Colonia Güell, (Dissertation), Mitteilungen des Instituts für leichte Flächentragwerke (IL), Bd 34, Stuttgart 1989)
G6. Church design. Interpretation of the church design as depicted in Gaudí's small hanging model. (Source: Tomlow, J.: Das Modell - Antoni Gaudís Hängemodell und seine Rekonstruktion - Neue Erkenntnisse zum Entwurf für die Kirche der Colonia Güell, (Dissertation), Mitteilungen des Instituts für leichte Flächentragwerke (IL), Bd 34, Stuttgart 1989)
G7. Colonia Güell church. Hanging model as a design tool, which was developed by Gaudí's team 1898-1908. The length scale was 1: 10, the weight scale 1:10.000. A paper decoration gives the model an aspect of volume. Das Modell, p. 66, OM 19. (Source: Courtesy of Cátedra Gaudí, Barcelona)
G8. Colonia Güell church. Hanging model interior view in an early design phase. Das Modell, p. 64, OM 2. (Source: Courtesy of Cátedra Gaudí, Barcelona)
G9. Colonia Güell church. Drawing by Gaudí on top of an inverted photograph, representing the exterior architecture. Das Modell, p. 67, OÜ 3a. The bleached state makes visible some saschets on the original photograph underneath. (Source: Courtesy of Cátedra Gaudí, Barcelona)
G10. Colonia Güell church. Drawing by Gaudí (Das Modell, p. 67, OÜ 3) on top of an inverted photograph, representing the exterior architecture. (Source: Courtesy of Cátedra Gaudí, Barcelona)
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G12. Colonia Güell church. Only the basement story with a crypt, to be considered as a unique piece of architecture, was built 1908 – 1914 on the base of the hanging model. (Source. Photo Jos Tomlow)
G13. Colonia Güell church. The reconstruction of Gaudí's hanging model by Rainer Graefe, Frei Otto, Jos Tomlow, Arnold Walz and team. The model was reconstructed May1982 - January 1983 for Kunsthaus Zürich. The reconstruction model is part of the Sagrada Familia museum. (Source: Photo Klaus Bach, courtesy of Institut für leichte Flächentragwerke, IL-Stuttgart)
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G18. Colonia Güell church. Interpretation of original model details. (Source: Tomlow, J.: Das Modell - Antoni Gaudís Hängemodell und seine Rekonstruktion - Neue Erkenntnisse zum Entwurf für die Kirche der Colonia Güell, (Dissertation), Mitteilungen des Instituts für leichte Flächentragwerke (IL), Bd 34, Stuttgart 1989)
G19. Colonia Güell church. A dynamometer. Gaudí measured the tension forces in hanging model strings with such a device. (Source: Rondelet, J.: Traité théorique et pratique de l'art de bâtir, Tome 3, Paris, p. 22, Pl. LXVIII, fig. 4)
G20. Colonia Güell church. Crypt plan with interpretation of suspension points of the hanging model. Measured drawing by Carlos Flores. (Source: Tomlow, J.: Das Modell - Antoni Gaudís Hängemodell und seine Rekonstruktion - Neue Erkenntnisse zum Entwurf für die Kirche der Colonia Güell, (Dissertation), Mitteilungen des Instituts für leichte Flächentragwerke (IL), Bd 34, Stuttgart 1989)
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G28. Colonia Güell church. Sketched reconstruction of the church design. (Source: Tomlow, J.: Das Modell - Antoni Gaudís Hängemodell und seine Rekonstruktion - Neue Erkenntnisse zum Entwurf für die Kirche der Colonia Güell, (Dissertation), Mitteilungen des Instituts für leichte Flächentragwerke (IL), Bd 34, Stuttgart 1989)
G29. Colonia Güell church. Church interior in the reconstruction model, added with contour lines of building parts. (Source: Tomlow, J.: Das Modell - Antoni Gaudís Hängemodell und seine Rekonstruktion - Neue Erkenntnisse zum Entwurf für die Kirche der Colonia Güell, (Dissertation), Mitteilungen des Instituts für leichte Flächentragwerke (IL), Bd 34, Stuttgart 1989)
G30. Colonia Güell church. Church interior reconstruction, depicting Antoni Gaudí as a visitor. (Source: Tomlow, J.: Das Modell - Antoni Gaudís Hängemodell und seine Rekonstruktion - Neue Erkenntnisse zum Entwurf für die Kirche der Colonia Güell, (Dissertation), Mitteilungen des Instituts für leichte Flächentragwerke (IL), Bd 34, Stuttgart 1989
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G34. Sagrada Familia. Graphic statics of a vault section, resting on a nave's pillar. (Source: Sugrañes, D.: Disposició estática del Temple de la Sagrada Familia, in: Anuario de la Asociación de Arquitectos de Cataluña 1923)
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Gm3. Rubió Bellver. Reconstruction of the parish church in Son Servera, Mallorca (1910). One bay's depth approx. 5,20 m. The differentiated architecture with diaphragm arches seems to be studied with the help of graphic statics. Reconstruction by Reuter, Tomlow, Trautz-Fülöp, Neuss. (Source: Tomlow, J.: Reconstruction of the 1910 parish church project by Joan Rubió i Bellver in Son Servera, Mallorca, in: Gaudinismo - Projekte der Gaudí-Schüler Jujol und Rubió, Geschichte des Konstruierens VII, Konzepte SFB 230 Heft 43, Stuttgart Juli 1995)
Gm4. Jujol Gibert. Vistabella church, 1917-1923. Interpretation of the central tower as tangents of parabolic shaped arches. This assumption states Jujol's rational and structural relevant approach in this case. (Source: Tomlow, J.: Bericht über die Fertigstellung der Montferri-Kirche von Josep-Marìa Jujol i Gibert, in: Gaudinismo - Projekte der Gaudí-Schüler Jujol und Rubió, Geschichte des Konstruierens VII, Konzepte SFB 230 Heft 43, Stuttgart Juli 1995)
Gm5. Jujol Gibert. Façade drawing of Santuari de Montserrat, in Montferri, 1928. The arches are all shaped by the catenary. (Source: Courtesy of Arxiu Jujol, Flores, C.: Gaudí, Jujol y el Modernismo catalán, Madrid 1982)
Gm6. Jujol Gibert. Design drawing, section, of Santuari de Montserrat, Montferri, 1928-1930 (Source: Courtesy of Arxiu Jujol, Flores, C.: Gaudí, Jujol y el Modernismo catalán, Madrid 1982)
Gm7. Jujol Gibert. Plan of Santuari de Montserrat, Montferri, showing the rational lay out of vault sections. Drawing by Bassegoda Arquitectos. (Source: Tomlow, J.: Bericht über die Fertigstellung der Montferri-Kirche von Josep-Marìa Jujol i Gibert, in: Gaudinismo - Projekte der Gaudí-Schüler Jujol und Rubió, Geschichte des Konstruierens VII, Konzepte SFB 230 Heft 43, Stuttgart Juli 1995)
Gm8. Jujol Gibert. Building photo of Santuari de Montserrat. Around 1930 building activity stopped. (Source: Flores, C.: Gaudí, Jujol y el Modernismo catalán, Madrid 1982)
Gm9. Jujol Gibert. Santuari de Montserrat in Montferri. Reconstruction and finishing project by Joan Bassegoda Nonell, Josep-Marià Jané Coca, Josep-Marià Jané Casas, Jos Tomlow, on behalf of the Ayuntamiento de Montferri, 1989-1999. (Source: Photo Jos Tomlow)
Gm10. Jujol Gibert. Santuari de Montserrat in Montferri. (Source: Photo Jos Tomlow)
Gm11. Jordi Bonet Armengol. Sant Medir church in Barcelona, 1955, The balancing of hyperboloid parabolic shells of reinforced masonry, is studied with an inventive hanging model. (Source: Photo Jos Tomlow)
Gm12. Jordi Bonet Armengol. The hanging model for Sant Medir church in Barcelona, shown in inverted position. (Source: Courtesy of Jordi Bonet Armengol)
Gm13. Jordi Bonet Armengol. Sant Medir church in Barcelona, during building. (Source: Courtesy of Jordi Bonet Armengol)